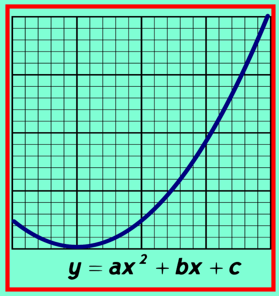
Math 125

Fall 2021

Lecture 46



Class QZ 35

Given $\frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1$

$a^2 = 9$, $a = 3$

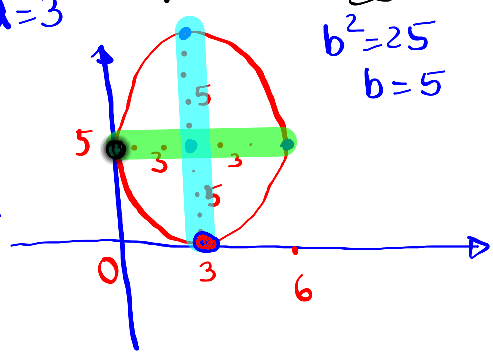
$b^2 = 25$

$b = 5$

Center

$(3,5)$ ✓

Draw ✓



Domain & Range

$[0,6]$ $[0,10]$

All intercepts

x-Int $(3,0)$ ✓

Y-Int $(0,5)$ ✓

find x & y

$$\begin{cases} x + y = 3 \\ x^2 + xy + y^2 = 7 \end{cases}$$

$x = 3 - y$

$(3-y)^2 + (3-y)y + y^2 = 7$

$9 - 6y + y^2 + 3y - \cancel{y^2} + \cancel{y^2} - 7 = 0$

$y^2 - 3y + 2 = 0$

$(y-2)(y-1) = 0$

$y = 2$ $\quad$ $y = 1$

$y = 2$ $\quad$ $x = 3 - 2 = 1$

$\boxed{(1,2)}$

$y = 1$ $\quad$ $x = 3 - 1 = 2$

$\boxed{(2,1)}$

$\{(2,1),(1,2)\}$

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y varies directly as x^4.

y is 128 when x is 2.

Find y when x is 3.

$y = Kx^4$

$128 = K \cdot 2^4$

$128 = K \cdot 16$

$\boxed{K = 8}$

$y = 8x^4$

$y = 8(3)^4$

$= 8(81) \Rightarrow \boxed{y = 648}$

y varies $\boxed{\text{inversely}}$ as $\sqrt{x}$.

y is 8 when x is 16.

Find y when x is 4.

$$y = \frac{K}{\sqrt{x}}$$

$$8 = \frac{K}{\sqrt{16}}$$

$$8 = \frac{K}{4}$$

$$\boxed{K = 32}$$

$$y = \frac{32}{\sqrt{x}}$$

$$y = \frac{32}{\sqrt{4}} = \frac{32}{2} \Rightarrow \boxed{y = 16}$$

z varies directly as x^2 and inversely as $\sqrt[3]{y}$.

When $x = 8$ and $y = 8$, z becomes 16.

Find z when $x = 12$ and $y = 64$.

$$z = \frac{Kx^2}{\sqrt[3]{y}}$$

$$16 = \frac{K \cdot 8^2}{\sqrt[3]{8}}$$

$$16 = \frac{K \cdot 64}{2}$$

$$32 = 64K$$

$$\boxed{K = \frac{1}{2}}$$

$$z = \frac{Kx^2}{\sqrt[3]{y}}$$

$$z = \frac{x^2}{2\sqrt[3]{y}}$$

$$z = \frac{12^2}{2\sqrt[3]{64}}$$

$$= \frac{144}{2 \cdot 4} = \frac{144}{8} = 18$$

$$\boxed{z = 18}$$

Use square-root method to Solve

1) $x^2 = 200$

$x = \pm\sqrt{200}$

$x = \pm\sqrt{100}\sqrt{2}$

$x = \pm 10\sqrt{2}$

$\{\pm 10\sqrt{2}\}$

2) $x^2 = -32$

$x = \pm\sqrt{-32}$

$= \pm\sqrt{16}\sqrt{2}\sqrt{-1}$

$= \pm 4\sqrt{2}\, i$

$\{\pm 4\sqrt{2}\, i\}$

Use square-Root method to Solve

1) $(2x-5)^2 = 98$

$2x - 5 = \pm\sqrt{98}$

$2x = 5 \pm \sqrt{49}\sqrt{2}$

$\boxed{x = \dfrac{5 \pm 7\sqrt{2}}{2}}$

$\left\{\dfrac{5 \pm 7\sqrt{2}}{2}\right\}$

2) $(4x+3)^2 = -20$

$4x + 3 = \pm\sqrt{-20}$

$4x = -3 \pm \sqrt{4}\sqrt{5}\sqrt{-1}$

$x = \dfrac{-3 \pm 2\sqrt{5}\, i}{4}$

$x = \dfrac{-3}{4} \pm \dfrac{\cancel{2}\sqrt{5}}{\cancel{4}_{2}}\, i$

$\boxed{x = \dfrac{-3}{4} \pm \dfrac{\sqrt{5}}{2}\, i}$

Making a Perfect-Square:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Ex:

$$x^2 + 10x + 5^2 = (x+5)^2$$

$$x^2 - 12x + (-6)^2 = (x-6)^2$$

$$x^2 + 18x + (9)^2 = (x+9)^2$$

$$x^2 - 5x + \left(\frac{-5}{2}\right)^2 = \left(x - \frac{5}{2}\right)^2$$

Make a perfect-Square!

$$\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$$

$$x^2 + \frac{3}{5}x + \left(\frac{3}{10}\right)^2 = \left(x + \frac{3}{10}\right)^2$$

$$\frac{1}{2} \cdot \left(\frac{-5}{2}\right) = \frac{-5}{4}$$

$$x^2 - \frac{5}{2}x + \left(\frac{-5}{4}\right)^2 = \left(x - \frac{5}{4}\right)^2$$

Solve

$$x^2 + 6x - 1 = 0$$

LHS cannot be Factored.

$$x^2 + 6x + \boxed{3^2} = 1 + 3^2$$

Make a Perfect-Square on LHS

$$(x + 3)^2 = 10$$

Use Square-Root method.

$$x + 3 = \pm\sqrt{10}$$

$$\boxed{x = -3 \pm \sqrt{10}}$$

$$\{-3 \pm \sqrt{10}\}$$

Completing the Square Method

Solve

$$x^2 - 8x + 4 = 0$$

LHS is not Factorable.

$$x^2 - 8x + \boxed{(-4)^2} = -4 + (-4)^2$$

Use completing the Square method to Solve.

$$(x - 4)^2 = -4 + 16$$

$$(x-4)^2 = 12$$

Now use SRM

$$x - 4 = \pm\sqrt{12}$$

$$x = 4 \pm \sqrt{4}\sqrt{3}$$

$$\boxed{x = 4 \pm 2\sqrt{3}}$$

$$\{4 \pm 2\sqrt{3}\}$$

$ax^2 + bx + c = 0$, $a \neq 0$

Quadratic Equation

$b^2 - 4ac$ discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

Given $2x^2 - 5x - 7 = 0$

$ax^2 + bx + c = 0$

$a = 2$ $b = -5$ $c = -7$

$b^2 - 4ac = (-5)^2 - 4(2)(-7) = 25 + 56 = \boxed{81}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{81}}{2(2)} = \frac{5 \pm 9}{4}$$

$x = \frac{5+9}{4} = \frac{14}{4} = \boxed{\frac{7}{2}}$ $x = \frac{5-9}{4} = \frac{-4}{4} = \boxed{-1}$

$\left\{-1, \frac{7}{2}\right\}$

Given $3x^2 + 5x - 8 = 0$

$ax^2 + bx + c = 0$

1) $a = 3$ $b = 5$ $c = -8$

2) Evaluate $b^2 - 4ac = (5)^2 - 4(3)(-8)$

$= 25 + 96 = 121$

3) $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-5 \pm \sqrt{121}}{2(3)} = \frac{-5 \pm 11}{6}$

$x = \frac{-5+11}{6} = \frac{6}{6} = \boxed{1}$ $x = \frac{-5-11}{6} = \frac{-16}{6} = \boxed{\frac{-8}{3}}$

$\left\{ \frac{-8}{3}, 1 \right\}$

Given $x^2 + 20x + 100 = 0$

1) $a = 1$ $b = 20$ $c = 100$

2) Evaluate $b^2 - 4ac = 20^2 - 4(1)(100) = \boxed{0}$

3) Solve using Quadratic Formula.

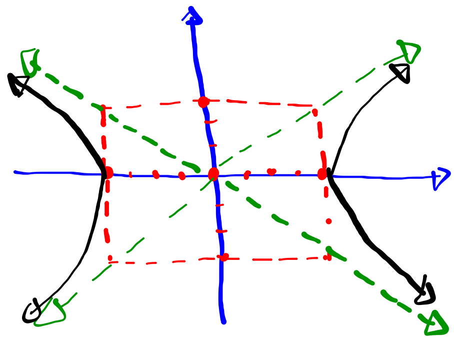
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-20 \pm \sqrt{0}}{2(1)} = \frac{-20 \pm 0}{2} = \frac{-20}{2} = \boxed{-10}$

$\boxed{x = -10}$ $\{-10\}$ Repeated Solution

Class QZ 36

Given $\frac{x^2}{16} - \frac{y^2}{9} = 1$

1) Center (0,0)



2) Draw a Complete Graph

3) Give All intercepts: x-Int $\boxed{\begin{matrix}(4,0)\\(-4,0)\end{matrix}}$ Y-Int $\boxed{\text{None}}$